In Fig. 4, trajectories and constant fuel contours are shown for comparison with the single-engine results of Fig. 3. It is apparent that substantial fuel penalties are associated with the two-engine concept.

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A New Concept in Rocket Engine Baffles

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Nomenclature

$R(x)$ = profile of baffle
t = time
x,y = transverse dimensions
$\epsilon Y(x)$ = width of cavity
ϵ = maximum width of cavity
η = translated y dimension
φ = velocity potential
Φ = amplitude of velocity potential
ω = frequency

Introduction

It is well known that the occurrence of transverse-spinning mode, high-frequency combustion instability in the liquid propellant rocket engine can often be eliminated by the use of baffles. These baffles are solid surfaces, which protrude from the injector plate, are perpendicular to it, and extend some determined distance into the rocket combustion chamber. The pattern or arrangement of these baffles required to prevent combustion instability cannot as yet be predicted a priori, and generally it may be said that such design is still an art rather than a science.

In some situations, however, there are certain rational guidelines to this design. It appears for some configurations that a design that segments the chamber into smaller sections will be successful in preventing the spinning mode in the chamber although the standing mode may still be present.[1] This is true even if the protrusion distance is rather small as compared to the chamber length provided that the baffle protrudes part of the way into a region of active combustion.

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If this is true, then segmenting a chamber by baffles changes the acoustic modes of oscillation of the full chamber to those permitted by the baffle cavities. Even standing modes can be altered in type, and in frequency of oscillation, from those permitted for the full chamber. This is important as larger and larger rocket engines are considered since, as a rocket engine size increases, the natural frequencies of the chamber decrease. However, it is well known that the higher the frequency, the more stable will be the engine.[2,3] This is caused by the frequency-dependent energy feedback to an acoustic wave from the combustion process. It appears that above a certain frequency (dependent upon propellant and injector type) this feedback is insufficient to sustain the wave by overcoming the damping caused by the exit nozzle. Therefore, if baffles really do alter the acoustic nature of the chamber in the manner described previously (note that there are some observed exceptions to this postulate), it is clear that a sufficiently large number of cavities can increase the frequencies of the allowable modes to a point where combustion instability is not possible in the transverse modes.

If this design principle is adopted, it is reasonable to ask if there is an optimum baffle pattern or shape. Of course, "optimum" must be defined. By this it will be meant that 1) the natural fundamental frequency of the cavities is as high as possible for any given number of cavities; 2) the baffles must not have a complicated shape that would entail manufacturing problems or cause excessive engine weight; and 3) there will be a dispersive device in the baffle shape so that, given an oscillation at the natural frequency of the cavity, the amplitude will be less than that at the point of maximum amplitude. It will be shown that, given the previously mentioned assumption concerning the effect of a baffle, a shape meeting the specifications does exist.

Analysis

The effect of the combustion process in determining the natural frequency of both longitudinal and transverse oscillations is of the order of the mean Mach number of the flow and is considered negligible as compared to unity. This would be a small correction even for large Mach numbers.[1,3] For transverse oscillations, variations in the longitudinal direction also are of the same order and are neglected here.[3] A linearized analysis gives a satisfactory first approximation to the natural frequency of oscillation within the cavity. The oscillation is described by the wave equation, and, if φ is the velocity potential, t is time, nondimensionalized by chamber diameter divided by speed of sound, and x and y are the transverse dimensions in a Cartesian system, nondimensionalized by chamber diameter, a solution assumed to be of the form

$$\varphi = \Phi(x, y)e^{i\omega t}$$

will be described by the Helmholtz equation

$$(\partial^2\Phi/\partial x^2) + (\partial^2\Phi/\partial y^2) + \omega^2\Phi = 0 \tag{1}$$

Now the frequency ω would be determined by the shape of the cavity by means of the application of boundary conditions to

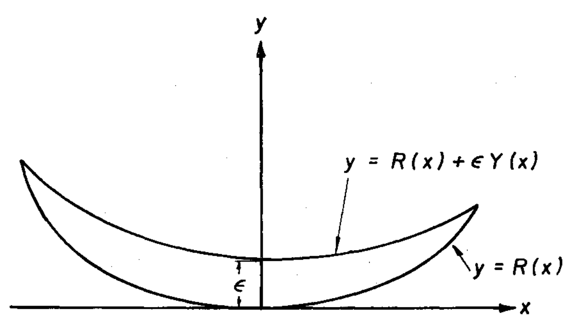

Fig. 1 The general geometrical configuration.

Eq. (1). The general class of cavities considered here is shown in Fig. 1. One cavity dimension is much larger than the other. The cavity may or may not be symmetric about the maximum-width line. $R(x)$ is the baffle profile on one side of the cavity. For convenience, $R(0)$ is taken as zero, and ϵ is defined as the maximum width so that $Y(0) = 1$. $\epsilon \ll 1$ is the important characteristic of this type of configuration. Note that ω in Eq. (1) is the physical frequency times chamber diameter divided by the speed of sound.

The lowest natural frequency of the cavity will be sought. It is clear that the expectation is that gradients in the y direction are much larger than those in the x direction. It is convenient, therefore, to scale‡ the y direction by the factor ϵ so that

$$x = \xi \qquad y = \epsilon\eta + R(x)$$

Under this transformation Eq. (1) becomes

$$\left[1 + \left(\frac{dR}{d\xi}\right)^2\right]\frac{\partial^2\Phi}{\partial\eta^2} + \epsilon^2\omega^2\Phi = -\epsilon^2\frac{\partial^2\Phi}{\partial\xi^2} + \epsilon\left[2\frac{dR}{d\xi}\frac{\partial^2\Phi}{\partial\eta\partial\xi} + \frac{d^2R}{d\xi^2}\frac{\partial\Phi}{\partial\eta}\right] \quad (2)$$

The boundary conditions, which state that the normal velocity at the baffle walls is zero, may be shown to be the following:

$$\left.\begin{aligned} &\left[1 + \left(\frac{dR}{d\xi}\right)^2\right]\frac{\partial\Phi}{\partial\eta} = \epsilon\frac{dR}{d\xi}\frac{\partial\Phi}{\partial\xi} \text{ along } \eta = 0 \\ &\left[1 + \left(\frac{dR}{d\xi}\right)^2\right]\frac{\partial\Phi}{\partial\eta} = \epsilon\frac{dR}{d\xi}\frac{\partial\Phi}{\partial\xi} + \epsilon^2\frac{dY}{d\xi}\frac{\partial\Phi}{\partial\xi} - \\ &\qquad \epsilon^2\left(\frac{dR}{d\xi}\right)^2\frac{dY}{d\xi}\frac{\partial\Phi}{\partial\xi} + 0(\epsilon^3) \text{ along } \eta = Y(\xi)\end{aligned}\right\} \quad (3)$$

If ω were of order unity, we find to the lowest order that Eq. (2) becomes $\partial^2\Phi/\partial\eta^2 = 0$ with the boundary conditions $\partial\Phi/\partial\eta = 0$ along $\eta = 0$ and $\eta = Y(\xi)$. This has the solution $\Phi = \Phi(\xi)$, which implies that the gradient in the y direction is less significant than the gradient in the x direction. This is contrary to our expectations. However, if ω were of the order $1/\epsilon$, the opposite would be true, i.e., $\partial^2\Phi/\partial\eta^2 = -\epsilon^2\omega^2\Phi/[1 + (dR/d\xi)^2]$ to the lowest order, and the gradient in the y direction would be larger than the gradient in the x direction. This indicates that ω is not of $0(1)$ but at least $0(1/\epsilon)$, either for a higher mode or an oscillation taking place in some small region characterized by a short length. And, in fact, the subsequent solution is found following the same method as presented in Ref. 5§

$$\Phi = \exp\left\{-\pi\left[\left(\frac{d^2R}{d\xi^2}(0)\right)^2 - \frac{d^2Y}{d\xi^2}(0)\right]^{1/2}\frac{\xi^2}{2\epsilon}\right\}\cos\pi\eta + 0(\epsilon) \quad (4)$$

where

$$\omega = (\pi/\epsilon) + 0(1) \quad (5)$$

This is an approximate solution. Consistent with the accuracy of the analysis, it should be mentioned that it is correct only up to the indicated precision and for x up to $0(\epsilon^{1/2})$. Regardless of how this is interpreted, as an oscillation in the x or y direction, it is obvious that the frequency is governed by the dimension ϵ rather than the full length. This result is typical of two-dimensional oscillations in regions with boundaries that have one characteristic dimension much smaller than the other. In that situation, the shorter dimension is considerably more significant in determining the frequency of the two-dimensional oscillation.

The physical interpretation of Eqs. (4) and (5) preferred here is that this represents an oscillation across the cavity rather than along it, and that it is the only type possible. This decay occurs because of the two dispersive mechanisms provided by the narrowing of the cavity away from the maximum-width line and by the curvature of the mean¶ line of the cavity. The difference in the curvatures (at the maximum-width line) of the two baffles surrounding the cavity is most critical in determining this decay rate. The larger the difference (or the larger the rate of "narrowing"), the greater the decay rate as shown by the factor $-d^2Y/dx^2(0)$ appearing in a square root in the exponential term. The additional dispersive mechanism results from the camber of the cavity section. This is shown by the appearance of the factor $[d^2R/dx^2(0)]^2$ in a square root in the exponential term. Therefore, one sees that the wave dispersive properties of this type of baffle can be controlled to some degree by adjusting the baffle shape.

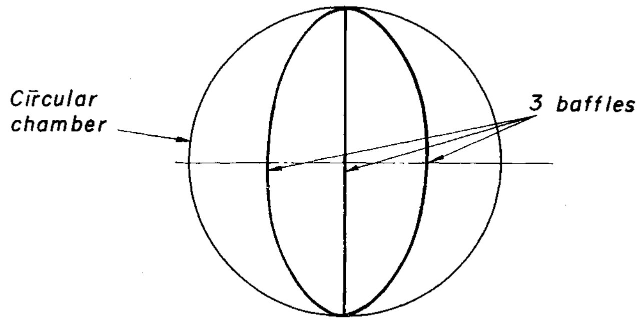

Fig. 2 A possible baffle configuration.

Possible Configuration

One possible shape for circular chambers is shown in Fig. 2 (assuming that three baffles are sufficient to raise the frequency of the acoustic mode to acceptable values). The concept could be extended readily to noncircular chambers, but so far it seems that this has no practical significance. The four cavities are chosen of equal width so that each has the same natural frequency, at least to the lowest order in ϵ. The difference in radii of curvature at the maximum-width points can be adjusted equally between the cavities or adjusted at will. The mean line curvature at the maximum-width point can also be adjusted, but generally it will always be greater for the outer cavities. The only constraint on the rest of the shape is that there is never another point at which the maximum width is equalled. There is, therefore, an infinity of shapes, which are theoretically equally acceptable. Of course, the analysis requires that $\epsilon \ll 1$, and how good the theo y is when ϵ is not too small can only be determined by experiment. Clearly, it will fail for $\epsilon = \frac{1}{2}$ since a well-known acoustic mode exists for the half-chamber if the full chamber is circular.

Discussion and Conclusions

Under the assumption that the purpose of a combustion chamber baffle is to segment a chamber into cavities with higher natural frequencies of gas oscillation than those of the full chamber, an interesting result from wave theory and its application has been presented. Baffle design criteria have been developed that are related to the specification of three quantities at the maximum-width point of each cavity, i.e.,

‡ This technique was originally applied to the problem of a long, slender vibrating membrane.[4]

§ A simplified version of Eq. (2) was solved in Ref. 5. In particular, terms of the order of $dR/d\xi$ and $d^2R/d\xi^2$ were neglected there but are included here.

¶ The mean line lies halfway between the two baffles, which form the cavity. Two transverse dimensions only are considered here.

this maximum width, the difference in the radii of curvature of the two cavity walls at this point, and the curvature of the mean line at this point. With this concept it is possible to maximize the acoustic frequencies allowable in the chamber, to do this with a smooth, somewhat arbitrary shape, and to provide a degree of dispersion, i.e., the amplitude decays with distance from the maximum-width point.

The maximization of frequency is known to have beneficial effects concerning combustion instability. The arbitrariness of shape away from the maximum-width point allows freedom concerning manufacturing and weight problems. The dispersive mechanisms are important in determining stability characteristics of an engine. However, even given the assumption concerning the baffle effect, it should be remembered that $\epsilon \ll 1$ is required, and the meaning of "very much less than" can only be determined experimentally. The theory is at best an asymptotic representation of the true state of affairs and should not be applied blindly. Since the allowable number of baffles in an engine is necessarily limited (the magnitude of ϵ is limited), care must be taken in interpreting experimental results.

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Pressure Orifice Shape Effect in Rarefied Flow with Heat Transfer

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Nomenclature

A = area
b = oblong orifice streamwise dimension
d = circular orifice diameter
d_{eq} = equivalent oblong orifice diameter
M_∞ = freestream Mach number
$\dot{q}$ = heat-transfer rate
p_i = pressure indicated or measured in orifice cavity
p_{i0} = pressure on surface outside the orifice
Re_∞ = freestream Reynolds number
T_0 = total temperature
T_w = wall temperature
λ_i = mean free path based on measured pressure p_i and wall temperature

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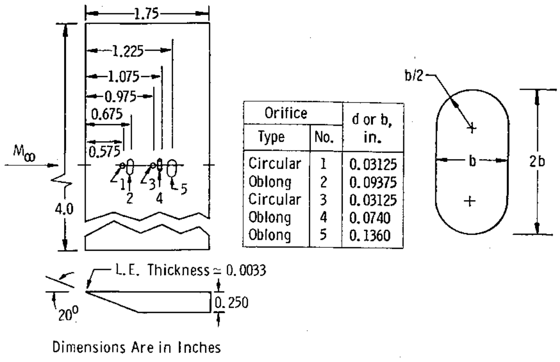

Fig. 1 Flat plate model and pressure orifices.

THE relation between the pressure in an orifice cavity and the true pressure on the surface of a cooled or heated body in rarefied, but not necessarily free-molecular, flow has been recently reported by Potter, Kinslow, and Boylan.[1] It was shown that, because of a form of nonequilibrium arising from unequal speed distributions that may exist between incoming and outgoing molecules in the orifice entrance region, the pressure sensed in the orifice cavity may be considerably in error.

Pressure data from impact-pressure probes and viscous interaction tests on flat plates and cones were used to illustrate these phenomena.[1] In addition, a semiempirical correction was derived by combining theoretical analysis for the limit of $d/\lambda_i \approx 0$ with experimental data obtained for cases where $d/\lambda_i \gg 0$.

All of the data described in Ref. 1 were obtained with circular-shaped orifices. A preliminary experimental investigation has been conducted to determine the effect of an oblong-shaped orifice on the corrections and thereby obtain a correction for pressures measured with oblong orifices.

The experiment was conducted in the von Karman Gas Dynamics Facility low-density wind tunnel (*L*). The model used was a sharp, flat plate, constructed of brass and water-cooled. The orifice locations and shapes are given in Fig. 1. Three different-sized oblong orifices and two circular orifices were used. The circular orifices were included to insure compatibility with a previous experiment. Based on thermocouple measurements, the plate surface temperature was about 300°K. The wind tunnel was operated at a free-stream Mach number of 10.15 and Reynolds number of 388/in. with a total temperature of about 3100°K.

The pressure over the plate clearly showed an influence of orifice size as predicted by Potter, Kinslow, and Boylan.[1] Data from the present experiment agreed well with previous measurements and, in addition, showed how the oblong orifice shape affected the results. The results are presented in Fig. 2.

Shown are data for circular and oblong orifices. The dimension d refers to the diameter of the circular orifice, and b refers to the streamwise dimension of the oblong orifice. The measured pressure p_i is ratioed to the true surface pres-

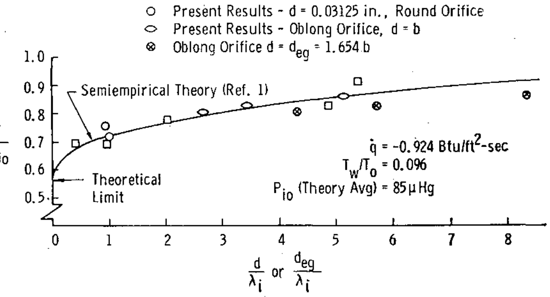

Fig. 2 Measured flat plate pressure in tunnel (*L*).